Understanding Rational and Irrational Numbers

**Understand** All numbers can be written with a decimal point. For example, you can rewrite $-2$ and $5$ with decimal points without changing their values.

$-2 = -2.0$ or $-2.00$ or $-2.000$, and so on

$5 = 5.0$ or $5.00$ or $5.000$, and so on

You can expand the decimal places of a number that already has digits to the right of the decimal point.

$-2.2 = -2.20$ or $-2.200$, and so on

$5.\overline{1} = 5.1\overline{1}$ or $5.11\overline{1}$, and so on

Each of the numbers above has a decimal expansion that ends either in zeros or in a repeating digit. Any number with a decimal expansion that ends in 0s or in repeating decimal digits is a **rational number**.

**Understand** Some numbers, like the ones below, do not end in 0s or in repeating decimal digits. The three dots, called an ellipsis, mean that digits continue, but not in a repeating pattern.

$\sqrt{2} = 1.41421\ldots$  $\sqrt{5} = 2.23606\ldots$  $-\sqrt{10} = -3.16227\ldots$

Any number with a decimal expansion that does not end in 0s or in repeating decimal digits is an **irrational number**. You have previously worked with a very important number, pi, which is represented by the symbol $\pi$.

$\pi = 3.14159\ldots$

The decimal expansion of $\pi$ does not end in 0s or in repeating decimal digits. It is an irrational number.

Every **real number** belongs either to the set of rational numbers or to the set of irrational numbers.
Connect

Is 0.07 rational or irrational?
Examine the digits to the right of the decimal point.
0.07 = 0.070 or 0.0700, and so on
0.07 is rational because its decimal expansion ends in 0s.

Is 3.45 rational or irrational?
Examine the digits to the right of the decimal point.
3.45 = 3.4545 or 3.454545, and so on
3.45 is rational because its decimal expansion repeats.

Is 10.049846... rational or irrational?
Examine the digits to the right of the decimal point.
10.049846...
10.049846... is irrational because its decimal expansion does not end in 0s or in repeating decimal digits.

Is \(\sqrt{8}\) rational or irrational?
Use a calculator to find the decimal form.
\(\sqrt{8} = 2.828427125...\)
Examine the digits to the right of the decimal point.
\(\sqrt{8}\) is irrational because its decimal expansion does not end in 0s or in repeating decimal digits.

How could you show that 4.95271 is a rational number using methods shown above?

DISCUSS
EXAMPLE A  Write each of the following rational numbers in fraction form.  
3, –0.9, 3.03

1  Express each number as a fraction of the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \neq 0 \). Use the place value of the rightmost digit to determine the value of the denominator.

The rightmost digit in 3 is in the ones place, so \( 3 = \frac{3}{1} \).
The rightmost digit in –0.9 is in the tenths place, so \( –0.9 = \frac{-9}{10} \).
The rightmost digit in 3.03 is in the hundredths place, so \( 3.03 = \frac{303}{100} \).

2  
\[
3 = \frac{3}{1} \quad -0.9 = \frac{-9}{10} \quad 3.03 = \frac{303}{100}
\]

EXAMPLE B  Convert the rational number 0.\( \overline{3} \) to a fraction.

1  Use algebra.
Set the number, 0.\( \overline{3} \), equal to \( n \).

\[
n = 0.\overline{3}
\]

There is one repeating digit, so multiply \( n \) by the first power of 10, or 10.

\[
10n = 3.\overline{3}
\]

2  Subtract the number, \( n \), from 10\( n \).

\[
\begin{align*}
10n &= 3.\overline{3} \\
- n &= 0.\overline{3} \\
9n &= 3
\end{align*}
\]

3  Solve the equation and simplify the result.

\[
\begin{align*}
\frac{9n}{9} &= \frac{3}{9} \\
n &= \frac{3}{9} \\
&= \frac{1}{3} \\
0.\overline{3} &= \frac{1}{3}
\end{align*}
\]

CHECK

How can you work backward from \( \frac{1}{3} \) to check the answer?
**Example C** Convert $0.\overline{45}$ to a fraction.

1. Use algebra. 
   Set the number, $0.\overline{45}$, equal to $n$.
   
   $n = 0.\overline{45}$
   
   There are two repeating digits, so multiply $n$ by the second power of 10, or 100.
   
   $100n = 45.\overline{45}$

2. Subtract the number, $n$, from $100n$.
   
   $100n = 45.\overline{45}$
   
   $\phantom{100n} - n = 0.\overline{45}$
   
   $\phantom{100n} \underline{99n} = 45$

3. Solve the equation and simplify the result.
   
   $\frac{99n}{99} = \frac{45}{99}$
   
   $n = \frac{45}{99}$
   
   $\phantom{n} = \frac{5}{11}$
   
   $0.\overline{45} = \frac{5}{11}$

**Discuss**

What steps could you use to express the decimal $0.\overline{83}$ as a fraction?
Practice

Identify whether the number is rational or irrational. Then explain why it is rational or irrational.

1. \(-101\)

2. \(\frac{8}{17}\)

3. \(21.192\)

4. \(\sqrt{7}\)

5. \(\sqrt{9}\)

6. \(\pi\)

7. \(\sqrt{50}\)

8. \(39.8\overline{1}\)

Write three equivalent decimal forms for each number.

9. \(19\)

10. \(21.5\)

11. \(-44.045\)

12. \(1.\overline{1}\)

If a square root has an integer value, is it rational?

REMEmBER Adding zeros to the end of a decimal does not change its value.
Lesson 1: Understanding Rational and Irrational Numbers

Complete each sentence.

13. −11.3 is rational because ________________________________.

14. \(\sqrt{19}\) is irrational because ________________________________.

15. 0.08\(\overline{3}\) is rational because ________________________________.

16. 2.1371938... is irrational because ________________________________.

Convert the repeating decimal to a fraction.

17. 0.\(\overline{6}\)

18. 1.\(\overline{1}\)

19. 4.\(\overline{4}\)

20. 9.\(\overline{09}\)

21. 2.\(\overline{90}\)

22. 4.\(\overline{54}\)

Choose the best answer.

23. Which is an irrational number?
   A. −3.3\(\overline{4}\)
   B. \(\sqrt{1}\)
   C. \(\sqrt{20}\)
   D. 11.2092

24. Which number is not equivalent to 13.02?
   A. 13.002
   B. 13.020
   C. 13.0200
   D. 13.020000

Solve.

25. Write Math Convert 3.1\(\overline{6}\) to a fraction. Explain your strategy or show the steps you used to convert the number.

26. Describe Describe two real-life applications of irrational numbers.